HW2 Solution

Wednesday, December 02, 2009

2:39 PM

(1) We will use MATLAB to find the values of N when i and j are between 0 and 7.

Here is the code:

```
[I J] = meshgrid(0:6,0:6);
N = I.^2 + I.*J + J.^2;
N = unique(reshape(N, 1, numel(N)));
N = N(N > 7);
N = N(1:15)
```

This part finds the unique values of N

Take only $N > 7$

Use only 15 values.

So, the next 15 values of N are

9	12	13	16	19	21	25	27	28	31	36	37	39	43	48

We know that we can't have any missing values of N between the above number because we have consider all i, j between 0 and 6. Any other values of N must come from (i, j) pair which has at least one of the i or $j \geq 7$ which will give $N \geq 7^2 = 49$.

(2) (a) Each simplex channel use 25 kHz.

So, each duplex channel use $25 \times 2 = 50$ kHz.

Total spectrum = 20 MHz

$$\#\text{ duplex channel} = \frac{20 \times 10^{6}}{50 \times 10^{3}} = \boxed{400 \text{ channels}}$$

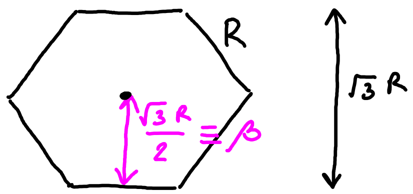
(b) Each cluster will use to whole 400 channels. These channels are divided among the cells in each cluster.

For $N=4$, there are 4 cells in a cluster.

Hence

$$\#\text{ channel} = \frac{400}{4} = \boxed{100 \text{ channels per cell site.}}$$

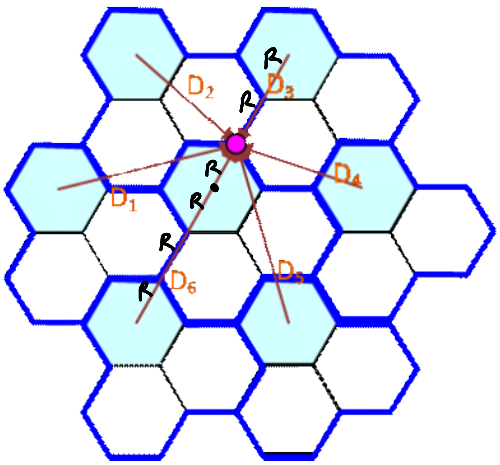
(3) (a) To find the distance $D_1, \ldots, D_6$, let's recall some facts about hexagon.



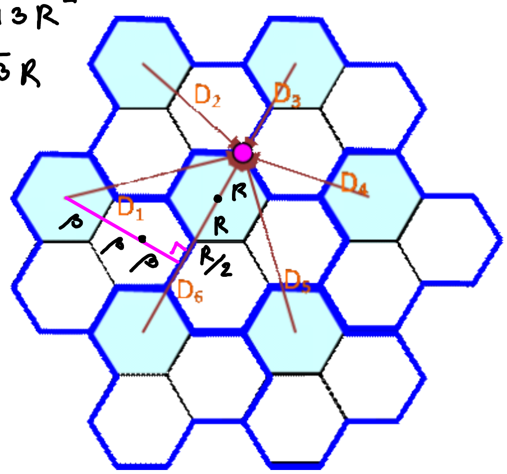
D_3 and D_6 are easy to find.

$$D_3 = R + R = 2R$$
$$D_6 = R + R + R + R = 4R$$

For the rest of the distances, the key to find them is to select suitable right triangles.

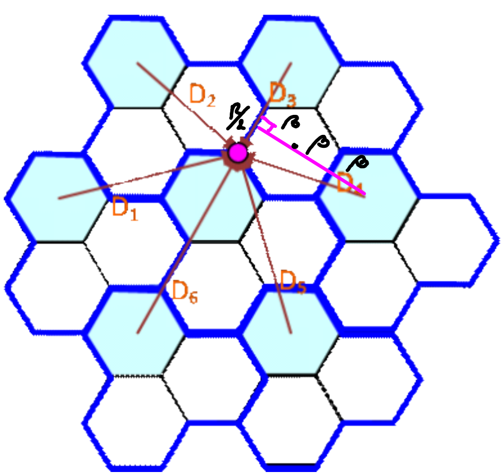


$$D_1^2 = D_5^2 = (3\beta)^2 + \left(\frac{5}{2}R\right)^2 = \left(9 \times \frac{3}{4} + \frac{25}{4}\right)R^2$$
$$= 13R^2$$
$$D_1 = D_5 = \sqrt{13}\,R$$



$$D_2^2 = D_4^2 = \left(\frac{R}{2}\right)^2 + (3\beta)^2 = \left(\frac{1}{4} + 9 \times \frac{3}{4}\right)R^2$$
$$= 7R^2$$

So, $D_2 = D_4 = \sqrt{7}\,R$



So,

$$D_1 = D_5 = \sqrt{13}\,R$$
$$D_2 = D_4 = \sqrt{7}\,R$$
$$D_3 = 2R$$
$$D_6 = 4R$$

(b) $$\frac{S}{I} = \frac{R^{-\gamma}}{\sum_{i=1}^{6} D_i^{-\gamma}} = \frac{1}{\sum_{i=1}^{6} \left(\frac{D_i}{R}\right)^{-\gamma}} = \frac{1}{2 \times (\sqrt{13})^{-4} + 2 \times (\sqrt{7})^{-4} + 2^{-4} + 4^{-4}}$$

$$= \boxed{\begin{array}{l} 8.399 \\ 9.242 \text{ dB} \end{array}} = 10 \log 8.399 \text{ dB}$$

(c) $\frac{D}{R} = \sqrt{3N} \Rightarrow D = \sqrt{3N} \times R = \sqrt{3 \times 3}\, R$ ($N = 3$)

$$= \boxed{3R}$$

$$\frac{S}{I} = \frac{1}{6 \times 3^{-4}} = \boxed{\begin{array}{l} 13.5 \\ 11.303 \text{ dB} \end{array}} = 10 \log 13.5 \text{ dB}$$

(d) In part (c) we use approximated distances and hence the answer is different from part (b) which use the exact distances.

When N is large, the difference will be small.

(4) $\frac{S}{I} = \frac{1}{K}\left(\sqrt{3N}\right)^4 = \frac{1}{K}(3N)^2 = \frac{1}{K} 9N^2$

We need this number to be $\geq 15 \text{ dB} = 10^{\frac{15}{10}} = 10^{\frac{3}{2}}$

Recall that we want N to be small to get large capacity value. Hence, we need to pick minimal value of N such that the above inequality is still satisfied.

For (a), we use $K = 6$.

$$\frac{S}{I} = \frac{1}{\cancel{6}_{2}} \times \cancel{9}^{3} \times N^2 \geq 15 \text{ dB}$$

$$N^2 \geq \frac{2}{3} \times 10^{\frac{3}{2}}$$

Possible values of N are 3 4 7 9 ....

$$N \geq \sqrt{\frac{2}{3} \times 10^{3/2}} = 4.591$$

From Q_1, the min value of N such that it is still ≥ 4.6 is $\boxed{N = 7.}$

For (b), we use $K = 2$.

$$\frac{S}{I} = \frac{1}{2} \times 9N^2 \geq 10^{3/2}$$

$$N \geq \sqrt{\frac{2}{9} \times 10^{3/2}} = 2.651$$

From Q_1, the min value of N such that it is still ≥ 2.6 is $\boxed{N = 3}$

is still ≥ 2.6 is $\boxed{N = 3.}$

For (c), we use $K = 1$

$$\frac{S}{I} = 9N^2 \geq 10^{3/2}$$

$$N \geq \sqrt{\frac{1}{9} \times 10^{3/2}} = 1.874$$

From Q1, the min value of N such that it is still ≥ 1.874 is $\boxed{N = 3}$

So, by using 120° sectoring, the capacity of the system increases from the case of omnidirectional antenna.

However, if we've already use 120° sectoring, using 60° sectoring does not help in term of capacity!!

(5) Let $\text{ErlangB}(m, A) = \dfrac{A^m/m!}{\sum_{i=0}^{m} A^i/i!}$

This gives the probability of blocking (P_b).

Ofcourse, we want P_b to be small.
In this question, we want $P_b \leq \frac{0.5}{100} = 0.005$.

For fixed m, ErlangB(m, A) is an increasing function of A. Hence, if we don't want P_b to be greater than some value, we will need to limit the value of A to be less than some max quantity as well.

(a) $m = 5 \Rightarrow P_b = \text{ErlangB}(5, A) \leq 0.005$

$$\big\downarrow \text{MATLAB}$$

$$A \leq 1.13 \text{ Erlangs}$$

Each user generates 0.1 Erlangs.

So N users will generate $N \times 0.1$ Erlangs.

Hence, we need $N \times 0.1 \leq 1.13$

$$N \leq 11.3$$

So, the system can support $\boxed{11 \text{ users}}$

(b) $m = 15 \Rightarrow$ Erlang B $(15, A) \lesssim 0.005$

$$A \lesssim 7.38$$

$$\Rightarrow N \lesssim 73.8$$

So, the system can support $\boxed{73 \text{ users}}$

(c) $m = 25 \Rightarrow$ ErlangB $(25, A) \lesssim 0.005$

$$A \lesssim 15$$

$$\Rightarrow N \lesssim 150$$

So, the system can support $\boxed{150 \text{ users}}$

6. $\lambda = 3$ calls per hour

$\frac{1}{\mu} = 5$ minutes $= \frac{5}{60}$ hour $= \frac{1}{12}$ hour.

(a) $A_u = \frac{\lambda}{\mu} = 3 \times \frac{1}{12} = \boxed{\frac{1}{4} \text{ Erlang}}$ per user

(b) Erlang B $(1, A) \lesssim 0.01$

$$\Rightarrow \underbrace{A}_{= N \times A_\mu} \lesssim 0.01$$

$$N \lesssim \frac{0.01}{1/4} = 0.04$$

So, the system can support $\boxed{0 \text{ user}}$

Note that this calculation comes from our assumption of M/M/m/m queue which assumes "infinite" number of users with extremely small Erlang per user.

Of course, we never have "infinite" number of users in real system.

So, it will not give an accurate answer for this case.

Intuitively, the system which has one channel should be able to support at least 1 user with 0% blocking.

The Erlang B formula should become more accurate when there are a lot of users.

(c) Erlang B $(5, A) \lesssim 0.01$

(c) $\text{ErlangB}(5, A) \leq 0.01$

$$\Rightarrow \underset{N \times A_u}{\overset{\shortparallel}{A}} \leq 1.36$$

$$N \leq 4 \times 1.36 = 5.44$$

so, the system can support $\boxed{\text{5 users}}$

(d) $\text{ErlangB}\left(5, 2 \times 5 \times \frac{1}{4}\right) = 0.0697 = \boxed{6.97\%}$

7 See another document.